Lesson 11-1: Parabolas

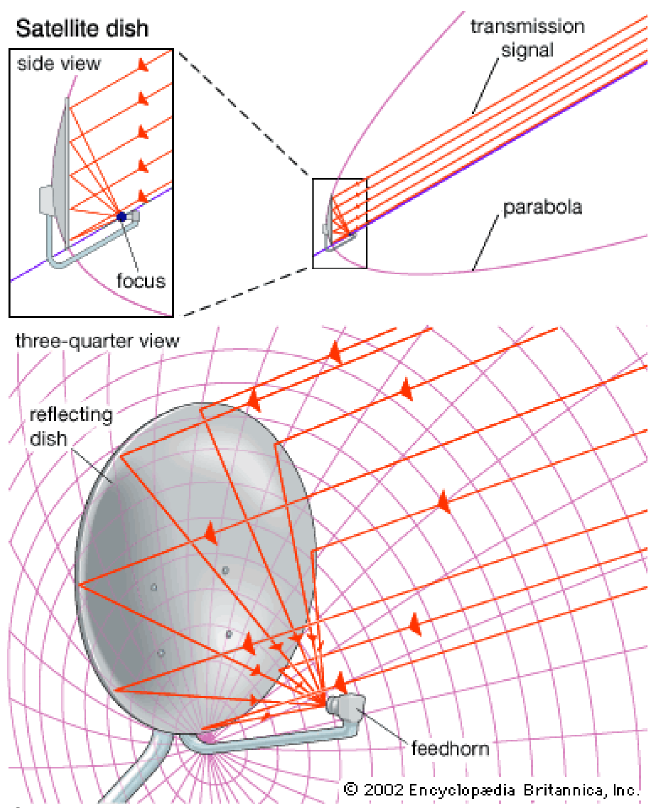
The first conic curve we will work with is the parabola. You may think that you've never really used or encountered a parabola before. Think about it…how many times have you been going somewhere and heard someone say "oh look, there's another parabola!" If you're like me, you've heard that, oh maybe about zero times.

Okay, so we really aren't aware of parabolas around us, but (if you take my word for it) they *are* there. Let's start by asking what is this a picture of?

Duh Mr. T, it's an antenna dish. Super, what could you tell me about the shape of the dish? Yes, it is curved, but it is a special curve. This is a practical example of a parabola in real-life use.

More accurately, this is what we call a ***parabolic*** dish antenna. The dish is in the shape of a parabola.

Your cell phone depends on these things…have I got your attention now? ☺



The cool thing about a parabolic dish is that it focuses into one point every signal that hits it straight on.

Do you know what the knob in the middle is used for? You could think of it as a collector. It is placed at the focus point for the parabolic dish, and it collects the entire focused signal and sends it to the equipment that processes it. It is often called a ***feedhorn***.

Here is another picture that shows this in better detail. The bright red lines represent the signal striking the surface of the dish. They are reflected to the focus point of the parabolic dish. At the focus point is the collector or feedhorn.

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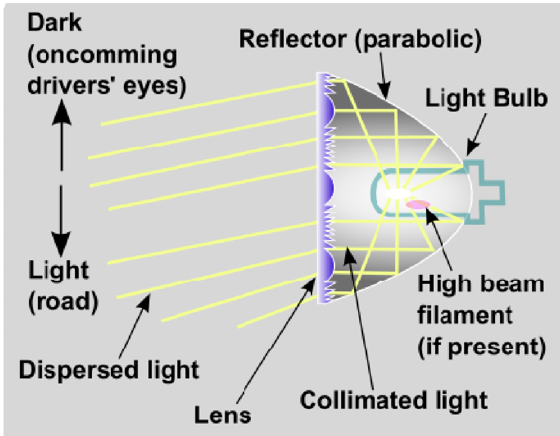
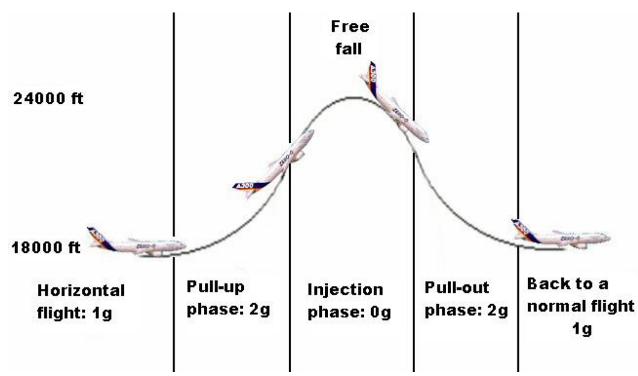
Cool, now what are some other things that parabolas are used for? Man, the list is ***huge***! Here are a few more just to give you an idea how parabolas are used:

- Car headlights



Solar energy

- Zero-gravity flight



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Where's the math?

Enough of the practical application silliness, let's get down to the real stuff…the ***math!*** Think back to chapter 5 and quadratic equations. There we discovered that the graph of a quadratic equation (a polynomial of degree 2) is a parabola.

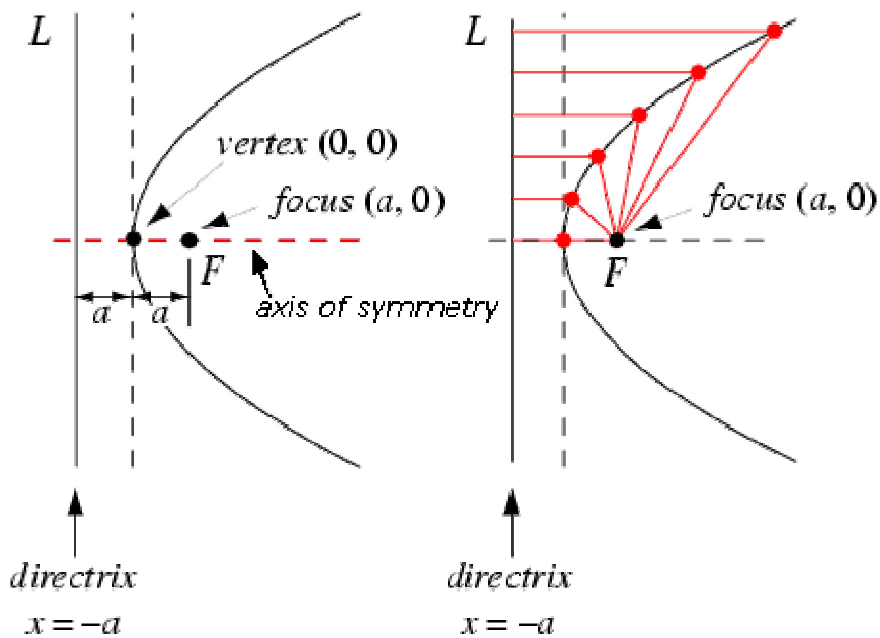
We just learned that a parabola focuses into one point, everything that hits it straight on. If you were going to build a parabolic dish antenna, or a light reflector, you'd need to be able to know where that focus point is so you could place the collector or the light source there. If you were off a little bit, it wouldn't work at all. So finding out where the focus point is for a parabola is very important!

In this lesson we will learn how to find the focus for a parabola.

The anatomy of a parabola

When we say that the parabola focuses everything that "hits it straight on" we are talking about coming in parallel to the ***axis of symmetry*** for the parabola.

The focus of a parabola lies on the axis of symmetry and is a very specific distance from the vertex of the parabola.



Behind the parabola is a straight line we call the ***directrix***. The directrix is perpendicular to the axis of symmetry. It together with the focus point actually creates the parabola.

A parabola is really the set of all the points that are equidistant from the focus and the directrix. If you look at the diagram above, on the right the red lines connect the red points on the curve of the parabola with the focus and the directrix. The distance from the point on the curve to the focus is the same as from the point to the directrix.

The vertex of the parabola is midway between the focus and the vertex. So if it is 5 units from the vertex to the focus, it is 5 units from the vertex to the directrix, but in the opposite direction.

Finding the focus given an equation

The parabolas that we will be working with in this lesson (and on the test!) are very simple: their vertex will always be the origin. This ***greatly*** simplifies the equation for the parabola.

We will be working with parabolas that are centered at the origin and either open straight up/down (vertical axis) or open left/right (horizontal axis). To find the focus, we will need to put the equation into a specific standard form:

	Equation	**Focus**	**Directrix**
Vertical axis	$x^2 = 4py$	(0, *p*)	*y* = -*p*
Horizontal axis	$y^2 = 4px$	(*p*, 0)	*x* = -*p*

If you're looking at the two versions of the standard equation, I'll bet you're asking "what's up with the 4 and the *p*?" Well, just to be clear, the *x* and the *y* are the variables. In front of the *y* is a number, a coefficient. Whatever that number is, if you factor 4 out (divide by 4), the remainder is *p.* The number *p* is the distance from the vertex to the focus and also to the directrix. In the table above, you can see that once you figure out what *p* is, you know where to place the focus.

Here are the steps you need to take to find the focus (and the directrix):

a. Put the equation in standard form:

    a. Get the squared variable by itself.

    b. Multiply the number by $\frac{1}{4}$ to get *p* and add 4 on the side.

b. Determine if the parabola has a vertical or horizontal axis:

    a. If vertical: the focus is (0, *p*) and the directrix is *y* = -*p*.

    b. If horizontal: the focus is (*p*, 0) and the directrix is *x* = -*p*.

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Writing a parabola's equation in standard form

Here are some examples to practice this on. This is basic algebra so with a little practice you should have this down. Write these parabolic equations in standard form:

1. $3y = 12x^2$ 2. $-3y^2 = x$ 3. $2x^2 = -\frac{1}{3}y$

Just follow the steps I listed above:

a. Get the squared variable by itself,

b. Multiply the number by $\frac{1}{4}$ to get *p* and add 4 on the side.

1. $3y = 12x^2$

$$3y = 12x^2$$
$$12x^2 = 3y \qquad \textit{Flip it around...}$$
$$x^2 = \frac{1}{4}y \qquad \textit{Get } x^2 \textit{ by itself}, \div \textit{both sides by } 6...$$
$$x^2 = 4\left(\frac{1}{4}\cdot\frac{1}{4}\right)y \qquad \textit{Multiply by } \frac{1}{4} \textit{ to find p...add the 4 outside}$$
$$x^2 = 4\left(\frac{1}{16}\right)y \qquad p \textit{ is } \frac{1}{16}, \textit{focus}\left(0, \frac{1}{16}\right), \textit{directrix } y = -\frac{1}{16}$$

2. $-3y^2 = x$

$$-3y^2 = x$$
$$y^2 = -\frac{1}{3}x \qquad \textit{Get } y^2 \textit{ by itself}, \div \textit{both sides by} -3...$$
$$y^2 = 4\left(\frac{1}{4}\cdot -\frac{1}{3}\right)x \qquad \textit{Multiply by } \frac{1}{4} \textit{ to find p...add the 4 outside}$$
$$y^2 = 4\left(-\frac{1}{12}\right)x \qquad p \textit{ is } -\frac{1}{12}, \textit{focus}\left(-\frac{1}{12}, 0\right), \textit{directrix } x = \frac{1}{12}$$

3. $2x^2 = -\frac{1}{3}y$

$$2x^2 = -\frac{1}{3}y$$
$$x^2 = -\frac{1}{6}y \qquad \textit{Get } x^2 \textit{ by itself, divide both sides by } 2...$$
$$x^2 = 4\left(\frac{1}{4}\cdot -\frac{1}{6}\right)y \qquad \textit{Multiply by } \frac{1}{4} \textit{ to find p...add the 4 outside}$$
$$x^2 = 4\left(-\frac{1}{24}\right)y \qquad p \textit{ is } -\frac{1}{24}, \textit{focus}\left(0, -\frac{1}{24}\right), \textit{directrix } y = \frac{1}{24}$$

Notice that the directrix uses the variable that *p* is with and is the negative of *p*.

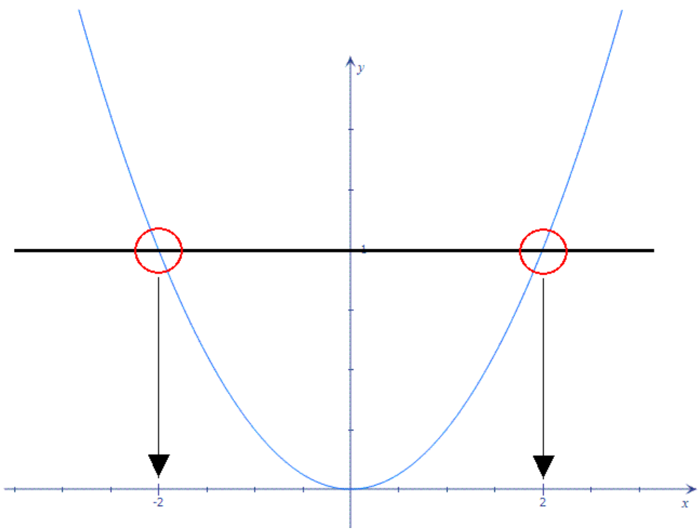
Determining if a parabola has a vertical or a horizontal axis

Now that we have the equation in standard from, we need to decide if it has a vertical or horizontal axis.

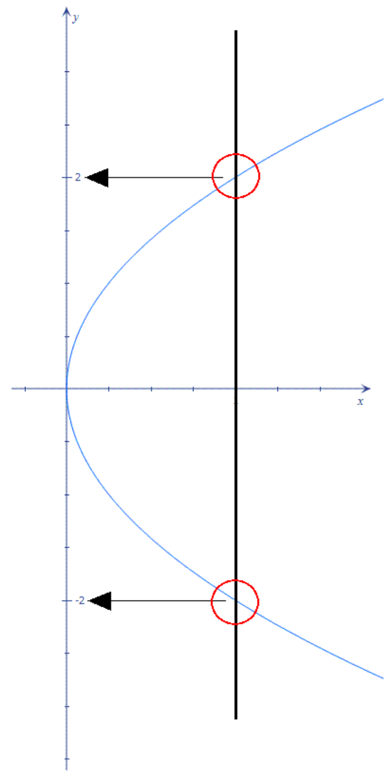
I suspect it may be hard to keep straight which equation has a vertical axis and which has a horizontal axis. This is the way I figure it out:

If you drew a line through the parabola, would the *x*'s have two values or would the *y*'s have two values?

For a vertical axis parabola
<u>the *x*'s have 2 values: x^2 = equation</u>.



For a horizontal parabola
<u>the *y*'s have 2 values: y^2 = equation</u>.



Here are some practice problems: determine if the equation is a vertical or horizontal axis parabola:

1. $3y = 12x^2$
2. $-3y^2 = x$
3. $2x^2 = -\frac{1}{3}y$
4. $\frac{1}{2}x = 4y^2$

Remember, if it is an x^2 equation, it is vertical. If it is a y^2 equation, it is horizontal.

1. Vertical
2. Horizontal
3. Vertical
4. Horizontal

Identifying the focus and directrix of a parabola

Just find *p*, and then determine if the parabola has a vertical or horizontal axis:

a. If vertical: the focus is (0, *p*) and the directrix is *y* = -*p*.

b. If horizontal: the focus is (*p*, 0) and the directrix is *x* = -*p*.

Let's try this with the 3 equations we put into standard form above.

1. $3y = 12x^2$ 2. $-3y^2 = x$ 3. $2x^2 = -\frac{1}{3}y$

Once we have the equation in standard form, *p* is obvious and we're good to go!

1. $3y = 12x^2$

$3y = 12x^2$	$x^2 = 4\left(\frac{1}{16}\right)y$	*Std form from above*
	$p = \frac{1}{16}$ & vertical axis	*...have* x^2
	$focus: \left(0, \frac{1}{16}\right)$	*Opens up...*
	$directrix: y = -\frac{1}{16}$	*...with directrix below*

2. $-3y^2 = x$

$-3y^2 = x$	$y^2 = 4\left(-\frac{1}{12}\right)x$	*Std form from above*
	$p = -\frac{1}{12}$ & horizontal axis	*...have* y^2
	$focus: \left(-\frac{1}{12}, 0\right)$	*Opens to the left...*
	$directrix: x = \frac{1}{12}$	*...with directrix to the right*

3. $2x^2 = -\frac{1}{3}y$

$2x^2 = -\frac{1}{3}y$	$x^2 = 4\left(-\frac{1}{24}\right)y$	*Std form from above*
	$p = -\frac{1}{24}$ & vertical axis	*...have* x^2
	$focus: \left(0, -\frac{1}{24}\right)$	*Opens down...*
	$directrix: y = \frac{1}{24}$	*...with directrix above*

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There is a point to note that will make it easier to match an equation with its graph. You can tell which way the parabola opens by looking at *p*:

- If *p* is positive, the parabola opens up (vertical) or to the right (horizontal).
- If *p* is negative, the parabola opens down (vertical) or to the left (horizontal).

Writing an equation given the focus

This is actually not as hard as it sounds. If you think about it, given the focus, you will know what the value *p* is just by looking at the focus. You can also tell if it has a vertical or horizontal axis:

1. First look at the coordinate place the *p* is in:
   - If it is the *x* coordinate (*p*, 0), the axis is horizontal and we have y^2 =
   - If it is the *y* coordinate (0, *p*), the axis is vertical and we have x^2 =
2. Second, just plug *p* into the right equation.

Here are two practice questions…write the standard form equation of the indicated parabola with its vertex at the origin:

1. Focus at (-4, 0) … horizontal axis, *p* = -4 … $y^2 = 4(-4)x \text{ or } y^2 = -16x$
2. Focus at (0, 3) … vertical axis, *p* = 3 … $x^2 = 4(3)y \text{ or } x^2 = 12y$

Writing an equation given the directrix

The directrix equation gives you *p*; just negate the number and you have *p*. The variable of the equation is the one that goes with *p*.

1. Directrix: *y* = 6 … *p* = -6, and x^2 = type … $x^2 = 4(-6)y \text{ or } x^2 = -24y$
2. Directrix: *x* = -1 … *p* = 1, and y^2 = type … $y^2 = 4(1)x \text{ or } y^2 = 4x$

A final parabolic point

You know, if we didn't have parabolas, we wouldn't have fast food. Yup, that's true! You don't believe me??? Just look at the next picture…you'll understand then! ;)